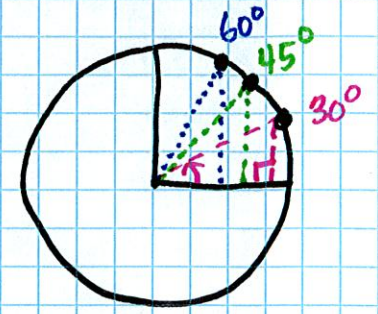
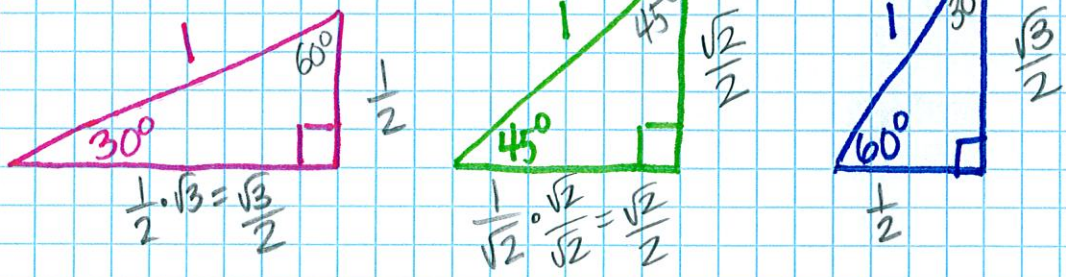
Notes: (Day 2) Finding Exact Trig. Values on Unit Circle



Special Right Triangles

$45°:45°:90°$
$x : x : x\sqrt{2}$

$30°:60°:90°$
$x : x\sqrt{3} : 2x$



$(\cos\theta, \sin\theta)$ (x, y)

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\tan 30° = \frac{\sin 30°}{\cos 30°} = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} \cdot \frac{2}{\sqrt{3}} = \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

$$\tan 45° = \frac{\sin 45°}{\cos 45°} = \frac{\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} = 1$$

$$\tan 60° = \frac{\sin 60°}{\cos 60°} = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} \cdot \frac{2}{1} = \sqrt{3}$$

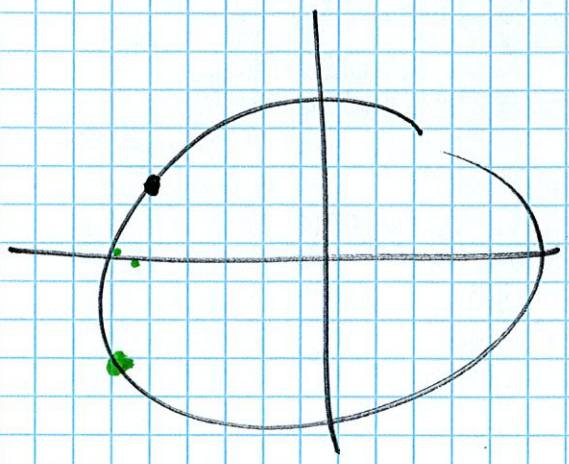
Reciprocal Identities:

$$\csc\theta = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{1}{\cos\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

$$\sin\theta = \frac{1}{\csc\theta} \qquad \cos\theta = \frac{1}{\sec\theta} \qquad \tan\theta = \frac{1}{\cot\theta}$$

- If you know one, you flip it to find the reciprocal function

Example(s)



A. $\cos\frac{5\pi}{4} = -\frac{\sqrt{2}}{2}$ (x-value)

B. $\sin(150°) = \frac{1}{2}$ (y-value)

C. $\sin\frac{7\pi}{6} = -\frac{1}{2}$ (y-value)

d. $\cos(0) = 1$ (x-value)

e. $\tan\frac{4\pi}{3} = \sqrt{3}$

f. $\tan 270° = ud$

g. $\csc\frac{11\pi}{6} = \left(\text{flip } \sin\frac{11\pi}{6}\right) = \frac{-2}{1} = -2$

h. $\cot 135° = \left(\text{flip } \tan 135°\right) = \frac{1}{-1} = -1$

i. $\sec\frac{\pi}{2} = \left(\text{flip } \cos\frac{\pi}{2}\right) = \frac{1}{0} = ud$

j. $\cot(-90°) = \left(\text{flip } \tan -90°\right) = 0$

K. $\sec(-405°)$ $-405° + 360 = -45° + 360° = 315°$ $\sec(315°) = \frac{2\sqrt{2}}{\sqrt{2}\sqrt{2}} = \frac{2\sqrt{2}}{2} = \boxed{\sqrt{2}}$ flip $\cos(315°)$

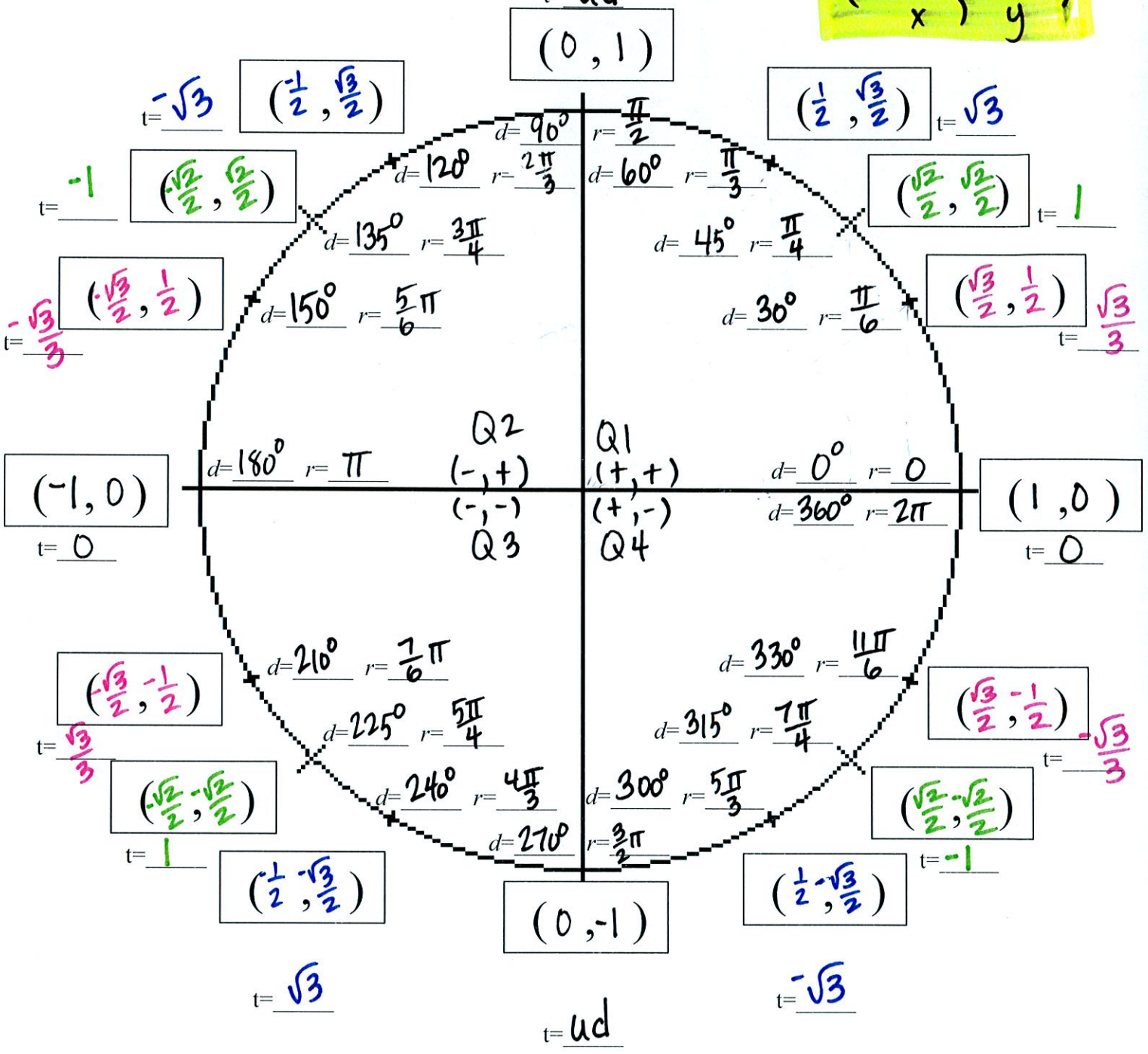
l. $\csc\left(\frac{29\pi}{6}\right)$ $\frac{29\pi}{6} - \frac{12\pi}{6} = \frac{17\pi}{6} - \frac{12\pi}{6} = \frac{5\pi}{6}$ $\csc\left(\frac{5\pi}{6}\right)$ flip $\sin\left(\frac{5\pi}{6}\right) = \frac{2}{1} = \boxed{2}$

m. $\cot\left(-\frac{\pi}{6}\right)$ $-\frac{\pi}{6} + \frac{12\pi}{6} = \frac{11\pi}{6}$ $\cot\left(\frac{11\pi}{6}\right)$ flip $\tan\left(\frac{11\pi}{6}\right) = \frac{-3}{\sqrt{3}}\frac{\sqrt{3}}{\sqrt{3}} = -\frac{3\sqrt{3}}{3} = -\sqrt{3}$

$(\cos\theta, \sin\theta)$
(x, y)

Unit Circle

t= ud



Reciprocal Identities

$\sin\theta = \frac{1}{\csc\theta}$ $\cos\theta = \frac{1}{\sec\theta}$ $\tan\theta = \frac{1}{\cot\theta}$

$\csc\theta = \frac{1}{\sin\theta}$ $\sec\theta = \frac{1}{\cos\theta}$ $\cot\theta = \frac{1}{\tan\theta}$

Tangent & Cotangent Identities

$\tan\theta = \frac{\sin\theta}{\cos\theta}$ $\cot\theta = \frac{\cos\theta}{\sin\theta}$

Pythagorean Identities

Double-Angle Formula